02/04/2020 Review notes

Exponent rules

- to multiply two powers with the same base, keep the base and add the exponents.
- to divide two powers with the same base, keep the base and subtract the exponents.
- to raise a power to a power, just multiply the exponents.

Practice examples:

1. $(-2x^2y)^2(x^{-4}y^3)$

$=(4x^4y^2)(x^{-4}y^3)$

$=4x^{4+(-4)}y^{2+3}$

$=\boxed{4y^5}$

2. $\left(\frac{3^2a^{-2}b^3}{ab}\right)^2$

$=\frac{9^2a^{-4}b^6}{a^2b^2}$

$=81a^{-6}b^4$

$=\boxed{\frac{81b^4}{a^6}}$

3. $\left(\frac{z}{y}\right)^{-1}\cdot\left(\frac{yz}{-2x}\right)^3$

$\left(\frac{z^{-1}}{y^{-1}}\right)\left(\frac{y^3z^3}{(-2)^3x^3}\right)$

$=\frac{y^3z^2}{-8y^{-1}x^3}$ $\quad 3-(-1)=4$

$=\boxed{\frac{y^4z^2}{-8x^3}}$

Modeling with exponential functions:

- Exponential functions are functions where a constant (called the base) is raised to a variable power.
- They have a general form of $\boldsymbol{f(x) = a \cdot b^x}$ OR $\boldsymbol{y = a \cdot b^x}$ (where b>0 and b ≠ 1).
- The exponential growth function has the form $y = a \cdot (1+r)^x$ where $a > 0$.
- The exponential decay function has the form $y = a \cdot (1-r)^x$ where $a > 0$.
- r is the growth/decay rate and must be in decimal form.

Modeling Examples:

4. The function $y = 3200(0.70)^t$ represents a model of:

A. exponential growth with 70% rate.
B. exponential decay with 70% rate.
C. exponential decay with 30% rate.
D. exponential growth with 30% rate.

$.7<1$ decay!

$1-r=.7$
$-1 \qquad -1$
$-r=-.3 \quad r=.3$
$=30\%$

5. To save for college, Amy deposited an amount of $20,000 at the beginning of 2020 in a savings account that pays 2% interest annually. How much will Amy have in her account by the end of 2029? 9 yrs

growth! $r=2\%=.02$

$y=a(1+r)^x \rightarrow y=20{,}000(1+.02)^x$

$=20{,}000(1.02)^9 \approx \boxed{\$23{,}901.85}$

Sketching an exponential function: $[y = a \cdot b^x]$

To graph an exponential function, you need to follow these steps:

- If $a > 0$, see if the function represents a growth or decay?
- Plot the y- intercept $(0, a)$ point.
- Plot a couple of points to the right of the y-intercept.
- Plot a couple of points to the left of the y-intercept.
- Connect the dots and extend the graph by knowing the end behaviors.
- If $a < 0$, expect an inverted graph and follow the previous steps again.

x	y
-2	8/9 or $.\overline{8}$
-1	4/3 or $1.\overline{3}$
0	2
1	3
2	9/2 or 4.5

Graphing example:

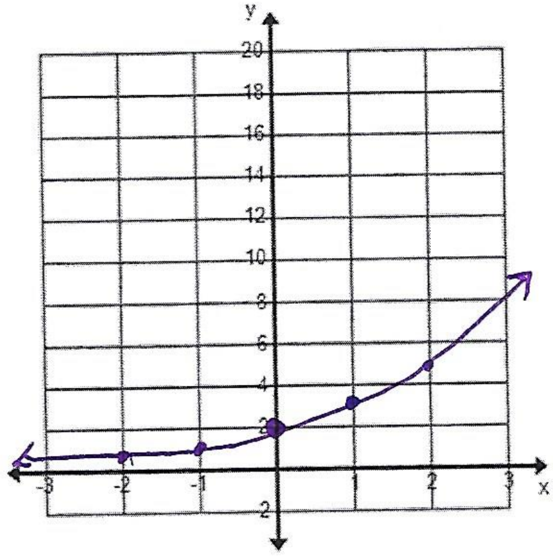
6. Graph the following exponential function:

$y = 2 \cdot \left(\frac{3}{2}\right)^x$ $\quad 3/2 > 1 = \text{growth}$

Increasing or Decreasing? increasing

Domain: $\mathbb{R}$ Range: $(0, \infty)$

End Behavior: As $x \to -\infty$, $y \to 0$
As $x \to \infty$, $y \to \infty$



Solving exponential equations:

- When solving exponential functions that don't have the same base, determine if the bases can be written as a common base, use the properties of exponents to simplify the problem. Once the bases are the same, then drop the bases, set the exponents equal to each other, and solve.

Examples:

Solve the following exponential equations:

7. $216^{-y} = \dfrac{1}{36^{2y+1}}$

$6^{3(-y)} = \dfrac{1}{6^{2(2y+1)}}$

$6^{3(-y)} = 6^{-2(2y+1)}$

$-3y = -4y - 2$
$+4y \quad +4y$

$\boxed{y = -2}$

8. $\dfrac{1}{512^{1-x}} = 8^{2x}$

$\dfrac{1}{8^{3(1-x)}} = 8^{2x}$

$8^{-3(1-x)} = 8^{2x}$

$-3(1-x) = 2x$

$-3 + 3x = 2x$
$-3x \quad -3x$

$-3 = -x$

$\boxed{x = 3}$

9. $11^{-2p+1} \cdot 121^{-p+3} = \dfrac{1}{11}$

$11^{-2p+1} \cdot 11^{2(-p+3)} = 11^{-1}$

$\underbrace{-2p + 1 + (-2p)}_{} + 6 = -1$

$-4p + 7 = -1$
$-7 \quad -7$

$\dfrac{-4p}{-4} = \dfrac{-8}{-4}$

$p = 2$

10. $\left(\frac{1}{8}\right)^{-2x+1} = \frac{1}{32}$

$2^{-3(-2x+1)} = 2^{-5}$

$6x - 3 = -5$
$+3 \quad +3$

$\dfrac{6x}{6} = \dfrac{-2}{6}$

$\boxed{x = \dfrac{-1}{3}}$